A Fair Share

Assessment Type	Recommended Grade Level	MYP Criterion Level
Real-life Application of Mathematics	Grade 10 (MYP5) Standard	MYP 5

MYP Assessment Criteria
Criterion D: Applying mathematics in real-life contexts

MYP Command Terms Used
identify, calculate, present, construct, compare, determine, explain, discuss, suggest, apply, justify, evaluate

MYP Global Context
Fairness and development

MYP Key Concept
Relationships

MYP Related Concepts
Models, Representation, Validity

MYP Branch of Mathematics
Reasoning with data

MYP Topics and Skills

- Percentages, proportions and financial calculations
- Cumulative income distributions and quintiles
- Constructing and interpreting Lorenz curves
- Calculating and interpreting the Hoover index
- Modelling income redistribution
- Evaluating the accuracy and limitations of mathematical models

Prior Knowledge Needed

- Calculating percentages of quantities
- Converting between percentages and decimals
- Calculating cumulative totals
- Plotting coordinates and drawing graphs
- Interpreting and comparing graphical data
- Distinguishing percentages from percentage points

Assessment Description

In this assessment, students investigate income inequality in two fictional countries by calculating cumulative income percentages, constructing Lorenz curves and determining Hoover indices. They use their findings to calculate the amount of income requiring redistribution to achieve perfect equality under a simplified model.

Students then investigate two alternative redistribution policies, comparing their effects on income distributions and Hoover indices. They discover that identical Hoover indices can represent different income distributions, before evaluating the accuracy, limitations and real-world implications of their mathematical models.

Materials Needed

Calculator, ruler, pencil and eraser, colored pencils for distinguishing Lorenz curves.

Task-specific instructions / Recommendations

Students should show working and support conclusions with mathematical evidence. Lorenz curves and the Hoover index are introduced in the task. All data are fictional, with equal incomes assumed within each quintile and unchanged total income following redistribution.

© The EdVaults – All Rights Reserved

Assessment Criterion D: *Applying mathematics in real-life contexts*

	Achievement Level Descriptor (MYP5)	Task Specific Descriptor
0	The student **does not** reach a standard described by any of the descriptors below.	
1-2	The student is able to: i. identify **some** of the elements of the authentic real-life situation ii. apply mathematical strategies to **find a solution** to the authentic real-life situation, **with limited success** iii. *(not demonstrated at this level)* iv. *(not demonstrated at this level)* v. *(not demonstrated at this level)*.	The student is able to: i. **identify** some relevant household and income data (Q1) ii. **apply** cumulative addition with limited success (Q2) iii. *(not demonstrated at this level)* iv. *(not demonstrated at this level)* v. *(not demonstrated at this level).*
3-4	The student is able to: i. identify the **relevant** elements of the authentic real-life situation ii. select, **with some success, adequate** mathematical strategies to model the authentic real-life situation iii. apply mathematical strategies to **reach a solution** to the authentic real-life situation iv. *(not demonstrated at this level)* v. **discuss** whether the solution makes sense in the context of the authentic real-life situation.	The student is able to: i. **identify** cumulative income shares under perfect equality (Q3) ii. **select** methods for constructing Lorenz curves and **find** Hoover indices (Q4, Q6a) iii. **apply** mathematics to **compare** distributions and **determine** and **interpret** Hoover indices (Q5, Q6b-c) iv. *(not demonstrated at this level)* v. **discuss** whether the Hoover indices agree with the Lorenz curves (Q7)
5-6	The student is able to: i. identify the **relevant** elements of the authentic real-life situation ii. select **adequate** mathematical strategies to model the authentic real-life situation iii. apply the selected mathematical strategies to **reach a valid solution** to the authentic real-life situation iv. **explain** the degree of accuracy of the solution v. **explain** whether the solution makes sense in the context of the authentic real-life situation.	The student is able to: i. **identify** the information needed to **calculate** redistribution amounts (Q8a) ii. **select** a suitable method for calculating redistribution amounts (Q8b) iii. **calculate** valid redistribution amounts using the selected method (Q8c) iv. **explain** the degree of accuracy of the redistribution estimates (Q9) v. **explain** whether the redistribution amounts make sense in context (Q10)
7-8	The student is able to: i. identify the **relevant** elements of the authentic real-life situation ii. select **appropriate** mathematical strategies to model the authentic real-life situation iii. apply the selected mathematical strategies to **reach a correct solution** to the authentic real-life situation iv. **justify** the degree of accuracy of the solution v. **justify** whether the solution makes sense in the context of the authentic real-life situation.	The student is able to: i. **identify** relevant information and conditions for **comparing** policies (Q11) ii. **select** and explain an appropriate strategy for comparing policies (Q12) iii. *correctly* **calculate** income shares, **construct** curves, and **determine** policy indices (Q13a-d) iv. **justify** the degree of accuracy of the policy Hoover indices (Q14) v. **justify** whether equal Hoover indices imply equally effective policies (Q15)

© The EdVaults – All Rights Reserved

Two fictional countries, Equitania and Prosperia, are reviewing the distribution of annual household income. Their governments want to understand the extent of income inequality and whether transferring income between household groups would make the distribution more equal.

Each country divides its households into five equally sized groups, ranked from lowest to highest household income. The table gives the percentage of all household income received by each group. Assume income is non-negative, and that the values describe each group accurately.

Household group	Equitania	Prosperia
Lowest 20%	8%	3%
Second 20%	12%	7%
Middle 20%	17%	12%
Fourth 20%	25%	23%
Highest 20%	38%	55%

For this mathematical model, assume that all households within each quintile receive the same income. In reality, individual household incomes within a quintile may differ.

A Lorenz curve plots the cumulative percentage of households (on the horizontal axis) against the cumulative percentage of income received (on the vertical axis). A straight line from $(0,0)$ to $(100,100)$ represents perfect equality. *For this task, we will connect consecutive plotted points with straight line segments.*

Part A: Investigating the distribution

(1) **Identify** two pieces of information from the situation that are *relevant* to *comparing income inequality*.

The five household groups are equal in size (20% each), and each group's percentage share of total household income is given. The order is from lowest to highest income.

[D: 1-2, i]

(2) **Calculate** the *cumulative* percentage of household income received by the lowest 20%, 40%, 60%, 80% and 100% of households in each country. **Present** your results in Table 1 below.

Cumulative Percentage of Households	Equitania	Prosperia
0%	0%	0%
20%	8%	3%
40%	20%	10%
60%	37%	22%
80%	62%	45%
100%	100%	100%

Table 1

[D: 1-2, ii]

© The EdVaults – All Rights Reserved

In a country where income is distributed equally among all households, the lowest 20% of households would receive 20% of the total income.

(3) **Identify** the *cumulative* percentage of total income that would be received by the lowest 40%, 60% and 80% of households *if income were distributed equally*.

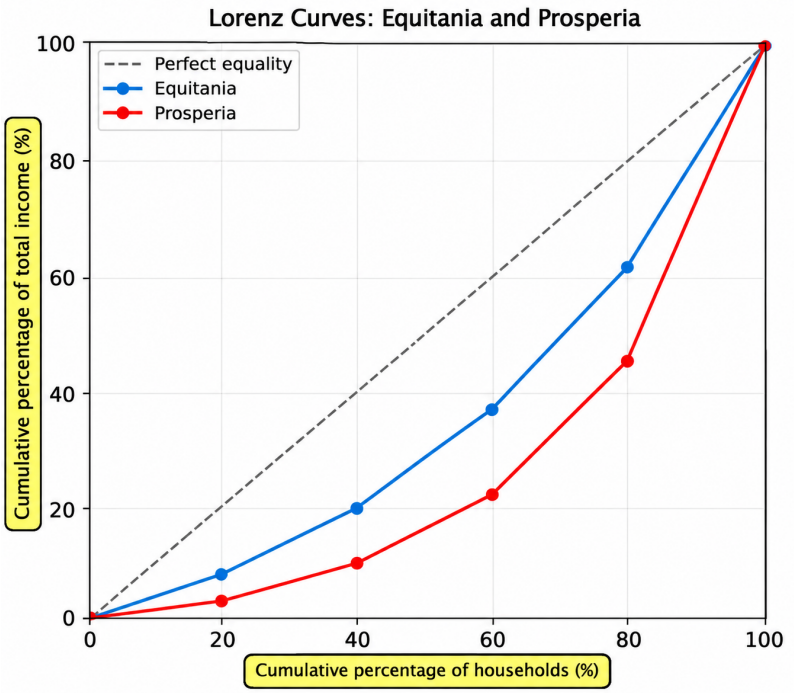
If income were distributed equally,

- *the lowest 40% of households would receive ___40%___ of the total income,*
- *the lowest 60% of households would receive ___60%___ of the total income,*
- *the lowest 80% of households would receive ___80%___ of the total income.*

[D: 3-4, i]

Graph 1 below shows the Lorenz curve for Equitania, together with the line of perfect equality.

(4) **Construct** the Lorenz curve for Prosperia on the same axes, using your cumulative percentages from Question (2). **Use** a ruler to draw straight line segments between consecutive points and clearly **label** your curve.



Graph 1

[D: 3-4, ii]

(5) **Compare** the income distributions of Equitania and Prosperia. *In your comparison, refer to the income received by the lowest-earning and highest-earning households, and use specific values from the table and Lorenz curves to support your conclusions.*

Prosperia has greater income inequality than Equitania. In Prosperia, the lowest 40% of households receive only 10% of the total income, compared with 20% in Equitania. Meanwhile, the highest 20% of households receive 55% of the total income in Prosperia, compared with 38% in Equitania.

Prosperia's Lorenz curve also lies further below the line of perfect equality than Equitania's, confirming that income is less equally distributed.

[D: 3-4, iii]

© The EdVaults – All Rights Reserved

Part B: Measuring inequality

The Hoover index measures income inequality by finding the greatest vertical distance between a country's Lorenz curve and the line of perfect equality. The vertical distance is calculated using:

$$\textbf{Vertical distance} = E - C$$

where E is the cumulative percentage of total income under perfect equality, and C is the actual cumulative percentage of total income for the country. For example, at 60% of households in Equitania, $E = 60$ and $C = 37$, and thus the vertical distance is $60 - 37 = 23$ percentage points.

(6)

a. **Calculate** the missing vertical distances for Equitania and Prosperia, and **write down** your answers in Table 2 below.

Cumulative Percentage of Households	Equitania: Vertical Distance	Prosperia: Vertical Distance
0%	0	0
20%	12	17
40%	20	30
60%	23	38
80%	18	35
100%	0	0

Table 2

[D: 3-4, ii]

b. **Determine** the Hoover index for each country using your results from part (a). **State** the cumulative percentage of households at which the greatest vertical distance occurs.

Equitania:
The greatest vertical distance is 23 percentage points, occurring at 60% of households.
Therefore, the Hoover index is 23%.
Prosperia:
The greatest vertical distance is 38 percentage points, occurring at 60% of households.
Therefore, the Hoover index is 38%.

[D: 3-4, iii]

c. **Explain** what the Hoover index of each country means *in terms of income redistribution*.

Equitania has a Hoover index of 23%, meaning that 23% of its total income would need to be redistributed from higher-income households to lower-income households to achieve perfect equality.

Prosperia has a Hoover index of 38%, meaning that 38% of its total income would need to be redistributed to achieve perfect equality.

[D: 3-4, iii]

© The EdVaults – All Rights Reserved

(7) **Discuss** whether your results are consistent with the income distributions shown by the Lorenz curves constructed in Question (4).

These results make sense because Prosperia's Lorenz curve lies further below the line of perfect equality than Equitania's. This indicates greater income inequality in Prosperia, so a larger percentage of income would need to be redistributed.

[D: 3-4, v]

Equitania's total annual household income is $400 billion, while Prosperia's is $500 billion.

(8)

a. **Identify** the information needed to determine the minimum annual household income that would need to be redistributed in each country to achieve perfect equality under the idealized model.

The relevant information is each country's Hoover index and total annual household income:

Equitania: 23% and $400 billion.

Prosperia: 38% and $500 billion.

[D: 5-6, i]

b. **Suggest** a mathematical method for using this information to calculate the amount of income that would need to be redistributed in each country.

Convert the Hoover index to a decimal, then multiply it by the country's total annual household income:

$$\text{Amount redistributed} = \frac{\text{Hoover Index}}{100} \times \text{Total Income}$$

[D: 5-6, ii]

c. **Calculate** the minimum amount of annual household income that would need to be redistributed in each country using your suggested method. **Show** your working.

Equitania: $0.23 \times 400 = 92$
Minimum redistribution: $92 billion.

Prosperia: $0.38 \times 500 = 190$
Minimum redistribution: $190 billion.

[D: 5-6, iii]

© The EdVaults – All Rights Reserved

Remember that for the mathematical model used in this task, we assume that all households within the same quintile receive an equal share of that quintile's income.

(9) **Explain** how accurate the calculated redistribution amounts are likely to be, considering that the income distributions are based on groups of households rather than individual household incomes.

The calculated amounts of $92 billion for Equitania and $190 billion for Prosperia are exact within the simplified model, assuming that all households within each quintile have the same income.

However, in reality, households within the same quintile may have different incomes. The grouped data do not show these differences, so the calculated Hoover indices may be lower than the actual values.

Consequently, the redistribution amounts should be treated as estimates of the minimum amounts required, rather than exact amounts for the real countries.

[D: 5-6, iv]

(10) **Explain** whether your calculated redistribution amounts for Equitania and Prosperia make sense, considering the Hoover indices and total annual household incomes of the two countries.

The calculated amounts make sense because Prosperia has both a higher Hoover index (38%) and a higher total annual household income ($500 billion) than Equitania, which has a Hoover index of 23% and a total annual household income of $400 billion.

Therefore, Prosperia would need to redistribute a larger amount of money ($190 billion) than Equitania ($92 billion). The amounts are consistent with the meaning of the Hoover index, since 38% of $500 billion is $190 billion and 23% of $400 billion is $92 billion.

[D: 5-6, v]

Part C: Testing redistribution policies

Prosperia's government can transfer $85 billion per year from the highest-earning 20% of households to households in other groups. The total national household income stays at $500 billion. It proposes two different policies:

Household group	Policy A: income received	Policy B: income received
Lowest 20%	$40 billion	$30 billion
Second 20%	$60 billion	$70 billion
Middle 20%	$85 billion	$85 billion
Fourth 20%	$125 billion	$125 billion
Highest 20%	$190 billion	$190 billion
Total	$500 billion	$500 billion

Table 3

© The EdVaults – All Rights Reserved

(11) **Identify** the relevant information and conditions needed to compare the effects of Policies A and B on income inequality in Prosperia.

Both policies redistribute $85 billion from the highest-income quintile, while the total household income remains $500 billion.

The policies allocate the transferred income differently between the two lowest quintiles. To compare their effects, the income shares must be converted into percentages and the revised Hoover indices determined.

[D: 7-8, i]

(12) **Suggest** a mathematical strategy for comparing the effects of Policies A and B on income inequality. **Explain** how your strategy would allow you to compare both the overall level of inequality and the income received by different household groups.

Convert the income amounts into percentages, calculate the cumulative income shares, and construct a Lorenz curve for each policy. Then determine the Hoover indices and compare the curves at different household percentages. This allows differences in income received across individual quintiles to be identified.

[D: 7-8, ii]

(13) **Apply** your suggested strategy to compare Policies A and B.

a. **Calculate** the percentage of total household income received by each quintile under both policies. **Write down** your answers in Table 4 below.

Household group	Policy A	Policy B
Lowest 20%	8%	6%
Second 20%	12%	14%
Middle 20%	17%	17%
Fourth 20%	25%	25%
Highest 20%	38%	38%

Table 4

[D: 7-8, iii]

© The EdVaults – All Rights Reserved

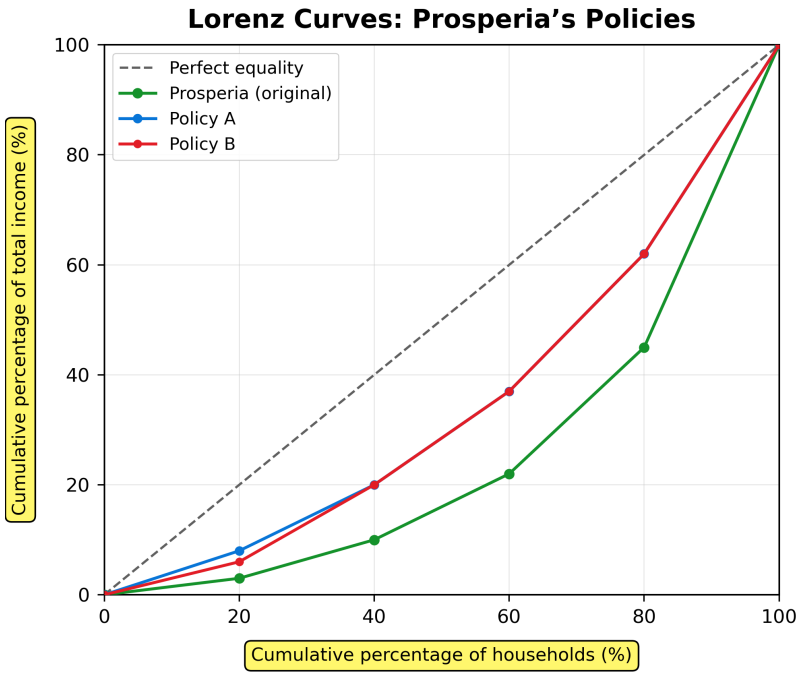
b. **Calculate** the cumulative percentage of total household income received by the lowest 20%, 40%, 60%, 80% and 100% of households under Policies A and B. **Write down** your results in the table below.

Cumulative Percentage of Households	Policy A: cumulative income (%)	Policy B: cumulative income (%)
0%	0	0
20%	8	6
40%	20	20
60%	37	37
80%	62	62
100%	100	100

Table 5

[D: 7-8, iii]

c. **Construct** the Lorenz curves for Policies A and B on Graph 2 below, which shows Prosperia's original Lorenz curve and the line of perfect equality. **Use** your results from part (b), **use** a ruler to draw straight line segments between consecutive points, and clearly **label** both curves.



Graph 2

Briefly **describe** where the two curves overlap.

The two curves overlap starting at 40% cumulative percentage of households.

[D: 7-8, iii]

© The EdVaults – All Rights Reserved

d. **Determine** the Hoover index for each policy, **showing** the vertical distances used to identify the greatest value. **Explain** what your results reveal about the two policies, considering the differences between their Lorenz curves.

Cumulative Households	Policy A: vertical distance	Policy B: vertical distance
20%	12	14
40%	20	20
60%	23	23
80%	18	18

Policy A has a Hoover index of 23%, and Policy B has the same. Both reduce Prosperia's original Hoover index from 38% by 15 percentage points.

Therefore, both policies reduce income inequality by the same amount according to the Hoover index.

Their curves differ near the lowest 20% of households, but this does not change the greatest vertical distance.

[D: 7-8, iii]

Part D: Is it a fair share?

(14) **Justify** the degree of accuracy of the Hoover indices calculated for Policies A and B as measures of income inequality. Refer to the information provided in the task and any assumptions made when constructing the Lorenz curves.

Both Hoover indices are exactly 23% in the simplified model. For each policy, the largest vertical distance occurs at 60% of households, where the cumulative income share is 37%, giving a distance of 60 − 37 = 23 percentage points.

However, the model assumes equal incomes within each quintile. Differences between individual households are not represented, so the true Hoover indices could be higher. Without individual household income data, the size of this possible difference cannot be determined.

[D: 7-8, iv]

© The EdVaults – All Rights Reserved

(15) **Evaluate** the following claim:

"Policies A and B are equally effective because they produce the same Hoover index."

Justify whether this conclusion makes sense in the context of income redistribution in Prosperia. **Use** mathematical evidence from both policies to support your answer.

Both policies reduce the Hoover index from 38% to 23%, so they are equally effective according to this measure. However, Policy A gives the lowest 20% of households 8% of total income, compared with only 6% under Policy B. Policy B instead gives more income to the second-lowest quintile.

Therefore, equal Hoover indices do not mean that the policies have identical effects. If the government's priority is to help the poorest households, Policy A is preferable.

The claim is reasonable when comparing only the Hoover indices, but not necessarily when considering how income is distributed among households.

Policy A's Lorenz curve lies above Policy B's between 0% and 40% of households, and the curves coincide afterward. Therefore, Policy A distributes income more equally, according to the Lorenz curve comparison, even though both policies have the same Hoover index.

[D: 7-8, v]

© The EdVaults – All Rights Reserved